TIM 125/225: LECTURE #19

Agenda:

1. Complete the discussion of HW#7, Prob 7 (Books-On-Line)

2. Aggregation for Safety Inventory
   HW #7, Prob #4

1. HW #7, Prob. #7
Books-on-Line

(a) Select feasible scenarios

	Warehouse Location Scenarios		
	East	Central	West
Current :→	0	0	1
	0	1	0
	1	1	1

(b) For each scenario determine relevant costs

Some of the costs, such as cycle inventory holding cost, are annual; while other costs (transportation) are weekly

Common basis: Convert all costs to a weekly basis

In the previous lecture we made a list of all relevant costs

(c) Calculate the total cost

Total cost = transportation cost + cycle inventory holding cost + safety inventory holding cost + fixed cost of the warehouse + variable cost of operating the warehouse

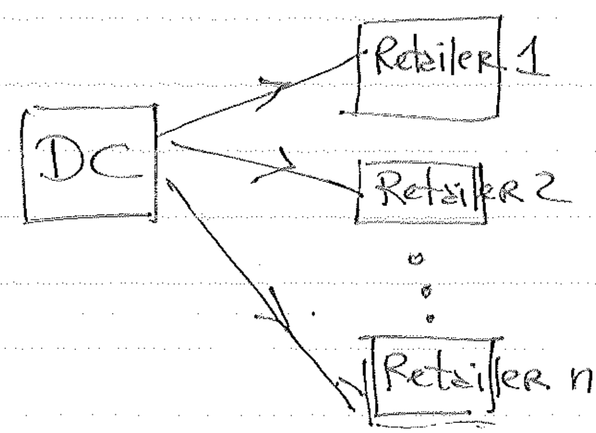
(d) Select the scenario that minimizes the total cost

Check: The total cost for each scenario is of the order of $100,000 [for weekly basis]

2. Aggregation

Aggregation has the following benefits

1. Demand forecasting at the distribution center (DC) is a lot more accurate than demand forecasting at the retailer



2. Aggregation reduces cycle inventory costs

We studied 3 cases: No aggregation
simple aggregation
tailored aggregation

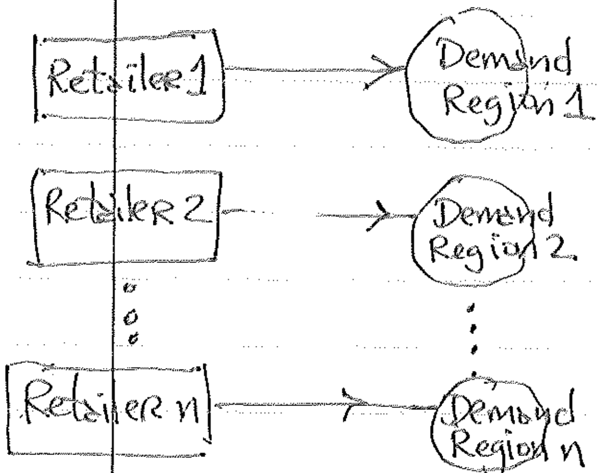
3. Aggregation reduces safety inventory costs

H.W. #7, Prob 4

(Ex 11.7 & 11.8 in the "Safety Inventory" chapter in the textbook

See next page: ⟶

Case A: No aggregation



(FOR Ex 11.7, n = 6)

For each region i: $[(D_L)_m]_i = L(D_W)_i$

$$(\sigma_{D_L})_i = \sqrt{L}(\sigma_W)_i$$

$$SS_i = (\sigma_{D_L})_i F^{-1}[(CSL)_{desired}]$$

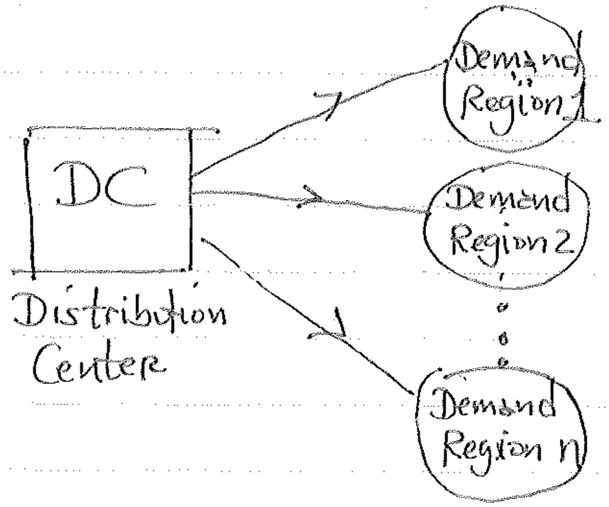
$$= \sqrt{L}(\sigma_W)_i F^{-1}[(CSL)_{desired}]$$

For n identical regions

$$(\sigma_W)_i \triangleq \sigma_W$$

Total SS for (n) regions $= n\sqrt{L}\,\sigma_W F^{-1}[(CSL_{desired})]$

Case B: Aggregation (DC)



For the DC

mean is additive:

$$[(D_L)_{mean}]_{DC} = L(D_W)_1 + L(D_W)_2 + \dots L(D_W)_n$$

$$[(D_L)_{mean}]_{DC} = \sum_{i=1}^{n} L(D_W)_i$$

variance is additive:

$$(\sigma_{D_L})^2_{DC} = (\sigma_{D_L})^2_1 + (\sigma_{D_L})^2_2 + \dots (\sigma_{D_L})^2_n$$

$$= L(\sigma_W)^2_1 + L(\sigma_W)^2_2 + \dots L(\sigma_W)^2_n$$

A : No aggregation

$$SS = n\sqrt{L}\,\sigma_{\overline{W}}\,F^{-1}\left[(CSL)_{desired}\right]$$

B : Aggregation

$$\left(\sigma_{\overline{D_L}}\right)^2_{DC} = \sum_{i=1}^{n} L\left(\sigma_{\overline{W}}\right)_i^2$$

$$\left(\sigma_{\overline{D_L}}\right)_{DC} = \sqrt{\sum_{i=1}^{n} L\left(\sigma_{\overline{W}}\right)_i^2}$$

$$(SS)_{DC} = \left(\sigma_{\overline{D_L}}\right)_{DC} F^{-1}\left[(CSL)_{desired}\right]$$

$$= \sqrt{\sum_{i=1}^{n} L\left(\sigma_{\overline{W}}\right)_i^2}\; F^{-1}\left[(CSL)_{desired}\right]$$

For n identical regions

$$\left(\sigma_{\overline{W}}\right)_i \triangleq \sigma_{\overline{W}}$$

$$(SS)_{DC} = \sqrt{\sum_{i=1}^{n} L\,\sigma_{\overline{W}}^2}\; F^{-1}\left[(CSL)_{desired}\right]$$

$$= \sqrt{n\,L\,\sigma_{\overline{W}}^2}\; F^{-1}\left[(CSL)_{desired}\right]$$

$$(SS)_{DC} = \sqrt{n}\,\sqrt{L}\,\sigma_{\overline{W}}\,F^{-1}\left[(CSL)_{desired}\right]$$

For n identical regions

$$\frac{(SS)_{\text{Case B} \to \text{Agg}}}{(SS)_{\text{Case A} \to \text{No Agg}}} = \frac{\sqrt{n}\sqrt{L}\,\sigma_W F^{-1}(\ )}{n\sqrt{L}\,\sigma_W F^{-1}(\ )}$$

$$= \frac{1}{\sqrt{n}}$$

For example for $n = 9$

$$\frac{(SS)_{Agg}}{(SS)_{No\ Agg}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$(SS)_{Agg} = \frac{1}{3}(SS)_{No\ Agg}$$

$\Rightarrow$ Aggregation of SS (for this case) reduces SS by 66%

Back to Ex 11.7 [Epson Printers]

Ex 11.7

Def of the problem ?

Effect of aggregation . . . .

Process

⋮

Implementation

Given Data : Table 11.6
for 6 countries [$(D_W)_i$ $(\sigma_{\overline{W}})_i$]
$i = 1, 2 \ldots 6$

Country	$(D_L)_i = L(D_W)_i$	$(\sigma_{\overline{D_L}})_i = \sqrt{L}(\sigma_{\overline{W}})_i$	$(\sigma_{\overline{D_L}})_i^2$
$i = 1$			
$i = 2$			
⋮			
$i = 6$			
			$\Sigma_i (\sigma_{\overline{D_L}})_i^2 =$

Aggregation

$$(\sigma_{D_L})^2_{DC} = (\sigma_{D_L})^2_1 + (\sigma_{D_L})^2_2 + \cdots (\sigma_{D_L})^2_6$$

$$(SS)_{DC} = (\sigma_{D_L})_{DC} \, F^{-1}\left[(CSL)_{desired}\right]$$

$(CSL)_{desired}$ ← 95% ≡ .95

Result $(SS)_{DC} = 21{,}000$

No Aggregation ;

County i	$(\sigma_{D_L})_i = \sqrt{L}(\sigma_W)_i$	$SS = (\sigma_{D_L})_i F^{-1}[(CSL)_{desired}]$
1	$(\sigma_{D_L})_1 =$	$(SS)_1 =$
⋮		
6	$(\sigma_{D_L})_6 =$	$(SS)_6 =$

$$\text{Total } SS = (SS)_1 + (SS)_2 + \cdots (SS)_6$$

Result = 48,400