- TIM 125/225
- LECTURE #12 (2/13/14)

Derivation of the equations for <u>Tailored Aggregation</u>

1. Aggregation for Cycle Inventory

3 cases (Multiple Products)

Case (a) No aggregation

Treated each product separately
(same as the single product)

Optimal Shipment frequency, $n_i^* = \sqrt{\dfrac{h_i C_i D_i}{2 S_i^*}} \rightarrow$ (1)(a)

$i = 1, 2, \ldots N$ (# of products)

What does h depend on?

- product life cycle
- size (space)

⋮

Case (b) Simple aggregation

Each shipment contains all products

$$S^* = S + \sum_{i=1}^{N} s_i$$

cost of each shipment — fixed cost — product specific cost

The optimal shipping frequency (common to all products)

$$n^* = \sqrt{\sum_{i=1}^{N} \frac{h_i C_i D_i}{2S^*}} \rightarrow 1(b)$$

Case (C): TAILORED Aggregation

Used when there are large differences in the demands of the various products

e.g. $D_1 = 10^4$; $D_2 = 10^6$; $D_3 = 10^2$

Questions

What are the relative shipping frequencies for each product?

Idea: When we know the relative frequencies, then we can allocate shipping costs for each product in a "fair" manner.

Procedure :

Step 1 : Treat each product as independent with respect to shipping (Case (a))

Calculate the shipping (ordering) frequency for product i,

$$\bar{n}_i \underset{1(a)}{=} \sqrt{\frac{h_i C_i D_i}{2(\underset{\text{fixed}}{S} + \underset{\text{variable}}{s_i})}} \rightarrow (2)$$

$i = 1, 2, \ldots N \triangleq$ total # of products

Determine the product k for which the shipping frequency is a maximum.

i.e. $\bar{n}_k \triangleq \max(\bar{n}_1, \bar{n}_2, \ldots \bar{n}_N) \rightarrow (3)$

Example if we have 3 products

$\bar{n}_1 = 30$, $\bar{n}_2 = 100$ $\bar{n}_3 = 12$

then $\bar{n}_k = \bar{n}_2 = 100$ & $k = 2$

Define $\underbrace{\bar{n} = \bar{n}_k}_{\text{shipping frequency of the most frequently ordered product}} \underset{(3)}{=} \max(\bar{n}_1, \bar{n}_2, \ldots n_N) \rightarrow (4)$

shipping frequency of
the most frequently ordered product

Step 2 :

Since each shipment will contain product k, let's allocate the fixed shipping cost S to product k

for product k, $$S_k^* = \underset{\text{fixed}}{S} + \underset{\text{variable}}{s_k} \longrightarrow (5)$$

for all other products, $i \neq k$, $$S_i^* = s_i \longrightarrow (6)$$
$(i = 1, 2, \ldots N; i \neq k)$

Step 3 :

Recalculate the order frequencies, $\bar{\bar{n}}_i$ for each product based on the shipping costs in (5), (6) Therefore $\bar{\bar{n}}_k \underset{1(a)}{=} \sqrt{\dfrac{h_k C_k D_k}{2 S_k^*}} \underset{(5)}{=} \sqrt{\dfrac{h_k C_k D_k}{2(S + s_k)}} \underset{(2)}{=} \bar{n}_k \underset{(4)}{=} \bar{n}$ $\longrightarrow (7)$

for all other products ($i \neq k$)

$$\bar{\bar{n}}_i \underset{1(a)}{=} \sqrt{\frac{h_i C_i D_i}{2 S_i^*}} \underset{(6)}{=} \sqrt{\frac{h_i C_i D_i}{2 s_i}} \longrightarrow (8)$$

$(i = 1, 2, \dots N;\ i \neq k)$

(The products are still being "independently shipped")

<u>Step 4</u>: Determine the relative shipping frequency, m_i, for each product

EQN (9) $\leftarrow$ $\bar{m}_i = \left[\dfrac{\text{(shipping frequency of the most frequently shipped product)}}{\text{shipping frequency of the } i^{\text{th}} \text{ product}}\right] \underset{(7,8)}{=} \dfrac{\bar{n}}{\bar{\bar{n}}_i} = \dfrac{\bar{n}_k}{\bar{\bar{n}}_i}$

$\bar{m}_i \geq 1 \quad (i = k,\ m_i = 1)$

Next

$$m_i \triangleq \lceil \overline{m}_i \rceil, \quad i = 1, 2, \ldots N \rightarrow (10)$$

(round-up to the nearest integer)

$$m_k = \lceil \overline{m}_k \rceil = 1 \text{ (most frequently ordered product)}$$

In our example,

$$\overline{m}_1 = \frac{100}{30} = 3.3; \quad \overline{m}_2 = \frac{100}{100} = 1; \quad \overline{m}_3 = \frac{100}{12} = 8.3$$

$$m_1 = \lceil \overline{m}_1 \rceil = 4, \quad m_2 = 1, \quad m_3 = \lceil 8.3 \rceil = 9$$

Step 5: Tailored Aggregation

Allocate shipping costs per order based on the relative shipping frequencies, m_i

$$\underset{\text{cost per shipment}}{S^*} = \underset{\text{fixed cost}}{S} + \frac{s_1}{m_1} + \frac{s_2}{m_2} + \cdots \frac{s_N}{m_N} \rightarrow (11)$$

Step 6 : Pose the optimization problem to minimize total costs (material cost + shipping cost + inventory holding cost) as in Case 2 or Case(b) [Simple aggregation]

$$\left[m_i = \frac{n_k}{n_i}\right] \Rightarrow n_i \underset{(9,10)}{=} \frac{\overset{\text{most frequent}}{n_k}}{m_i}, \quad (i = 1, 2, \dots N) \rightarrow (12)$$

$$(\text{total cost}) \; C_T \underset{(12)}{=} f(n_k)$$

$$\frac{dC_T}{dn_k} = 0 \Rightarrow \underbrace{n_k^*}_{\substack{\text{optimal shipping frequency}\\ \text{of the most frequently ordered}\\ \text{product}}} \underset{1(b)}{=} \sqrt{\sum_{i=1}^{N} \frac{h_i C_i D_i m_i}{2S^*}} \rightarrow (13)$$

where S^* is given by (11)

Shipping frequency, n_i^* for all other products $\underset{(12)}{=} \dfrac{n_k^*}{m_i} \longrightarrow (14)$ $(i = 1, 2, \ldots N)$

Optimal lot size $(Q_L)_i^* = \dfrac{D_i}{n_i^*} \longrightarrow (15)$ $(i = 1, 2, \ldots N)$

Use this procedure in

HW # 5, ~~Prob 4~~ Problem 3 (EX 10.3 Text) $\Rightarrow$ Ex 10.3 (Third Edition)

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done in HW#4 { (1) Separate Products
(2) Simple Aggregation
(3) Tailored Aggregation

How was eqn (13) for n_k^* obtained?

C_i' = Transportation cost + Inv holding cost

$$= (S^*)(n_k) + \sum_{i=1}^{N} \left(\frac{Q_L}{2}\right)_i h_i C_i$$

$$C_i' = \left(S + \sum_{i=1}^{N} \frac{s_i}{m_i}\right) n_k + \sum_{i=1}^{N} \frac{D_i}{2 n_i} h_i C_i$$

$$C_i' \underset{(12)}{=} \left(S + \sum_{i=1}^{N} \frac{s_i}{m_i}\right) n_k + \sum_{i=1}^{N} \frac{D_i m_i h_i C_i}{2 n_k}$$

$$\frac{dC_i}{dn_k} = \left(S + \sum_{i=1}^{N} \frac{s_i}{m_i}\right) - \sum_{i=1}^{N} \frac{D_i}{2 n_k^2} m_i h_i C_i = 0$$

$$n_k^* = \sqrt{\frac{\sum_{i=1}^{N} D_i h_i C_i m_i}{2\left(S + \sum_{i=1}^{N} \frac{s_i}{m_i}\right)}}$$